The ten kinds of conditionals and Chrysippus' criterion

Miguel López-Astorga

Institute of Humanistic Studies "Juan Ignacio Molina", University of Talca, Talca Campus (Chile). ORCID ID: 0000-0002-6004-0587. Email address: milopez@utalca.cl

Abstract:
The theory of mental models proposed ten possible interpretations for the conditional. In particular, Johnson-Laird and Byrne provided those interpretations in 2002. On the other hand, in ancient times, Chrysippus of Soli claimed that conditionals have to fulfill a requirement: the negation of their consequents must be inconsistent with their antecedents. This paper considers the ten interpretations of the conditional the theory of mental models indicates as ten different kinds of conditionals. Thus, it analyzes those ten types from Chrysippus' requirement. The results show that the ten kinds of conditionals that can be derived from the theses of the theory of mental models follow the criterion given by Chrysippus. Accordingly, it can be thought that the criterion refers to an essential characteristic of the conditional.

Keywords: Chrysippus' criterion; conditional; possibility; ; state-description; theory of mental models

Introduction

There is no doubt that a conditional such as (1) is hard to accept for people.

(1) If elephants can sing, then groundhogs can drive cars.

However, (1) is correct from the logical point of view. This is because standard logic assumes the material interpretation of the conditional. Following that interpretation, conditionals are true provided that a particular scenario does not happen: the scenario in which the antecedent or if-clause occurs and the consequent or then-clause does not hold. This means that (1) is true: while its consequent does not hold (groundhogs cannot drive) the antecedent is not true either (elephants cannot sing either).

Philo of Megara was who first proposed the material interpretation of the conditional (e.g., Bocheński, 1963; Mates, 1953; O'Toole & Jennings, 2004). This circumstance reveals that the problem is old. Nevertheless, the proposals to solve that problem also come from ancient times. That is the case of the solution offered by the Stoic philosopher Chrysippus of Soli. According to this solution, a conditional can only be deemed as conditional if it fulfills a requirement: the negation of its consequent should be incoherent with its antecedent (e.g., Barnes, Bobzien, & Mignucci, 2008; Gould, 1970; O'Toole & Jennings, 2004). Under this criterion, (1) would not be admissible: the fact that groundhogs cannot drive is not inconsistent with the fact that elephants can sing.

Chrysippus' criterion is relevant. It has been used to explain even contemporary cognitive issues (e.g., López-Astorga, 2016). For this reason, this paper will resort to it too. Nonetheless, the aim is different here: to gauge to what extent the diverse interpretations of the conditional proposed by a current framework are correct if Chrysippus' requirement is adopted. The framework is the one of the theory of mental models (e.g., Byrne & Johnson-Laird, 2020; Khemlani, Byrne, & Johnson-Laird, 2018Ragni & Johnson-Laird, 2020). This theory has claimed that ten interpretations of the conditional are possible for almost two decades (Johnson-Laird & Byrne, 2002).

Thereby, the present paper will have three parts. The first one will describe the ten interpretations of the conditional the theory of mental models has proposed. The second part will explain the logical formula that will be used to stand for Chrysippus' criterion (as shown below, Lenzen, 2019, was who actually introduced the formula). The final part will consider the ten interpretations to be kinds of conditionals and analyze them by means of the logical formula explained.

The conditional and its ten interpretations

This paper is not intended to present the main theses of the theory of mental models or to review its basic assumptions. However, some of its most important ideas are necessary to understand why the theory proposes ten interpretations for the conditional. The theory of mental models is a cognitive approach about the way human beings make inferences. But it has important linguistic dimensions. One of those dimensions is the interesting here.

That dimension refers to the fact that the theory relates every sentence to semantic possibilities in which that sentence can be true (see also, e.g., Johnson-Laird, 2012). Thus, the theory of mental models often attributes three possibilities to a sentence such as (1) (e.g., Khemlani et al., 2018):

(2) P(elephants sing & groundhogs drive) & P(elephants do not sing & groundhogs drive) & P(elephants do not sing & groundhogs do not drive).

Where 'P(X & Y)' stands for a possible scenario: the scenario in which both X and Y hold.

There is only one forbidden situation if (1) is true (see also, e.g., Johnson-Laird & Ragni, 2019):

(3) I(elephants sing & groundhogs do not drive).

Where 'I(X & Y)' represents an impossible scenario: the scenario in which both X and Y are the case.

However, this may not be a correct account; the possibilities can be different when the conditional is used in daily life. As pointed out, the theory is semantic, and it also acknowledges the role pragmatics plays. So, the meanings of the words and pragmatic circumstances can reveal the real possibilities corresponding to a particular sentence (see also, e.g., Quelhas, Johnson-Laird, & Juhos, 2010). As far as the conditional is concerned, the theory of mental models distinguishes until ten possible interpretations. They can be found in Table 4 in Johnson-Laird and Byrne (2002). That table offers examples of both factual and deontic conditionals. Only the factual ones will be considered here, since they suffice to make the point of the present paper. As it can be noted below,

to address the deontic ones would be trivial after analyzing the factual ones. This is because the results of doing that would be similar.[1]

The first interpretation is Tautology. The four combinations of clauses are possible for it. The example given by Johnson-Laird and Byrne is (4).

(4) "If there are lights over there then there may be a road" (Johnson-Laird & Byrne, 2002, p. 663).

Undoubtedly, (4) allows the four possible scenarios. They are indicated in (5).

(5) $P(p \ \& \ q) \ \& \ P(p \ \& \ \neg q) \ \& \ P(\neg p \ \& \ q) \ \& \ P(\neg p \ \& \ \neg q)$.

Where 'p' stands for the if-clause, 'q' represents the then-clause, and '¬' denotes negation.

Nonetheless, there are three interpretations that enable only three combinations. They are Conditional, Enabling, and Disabling.

The example of Conditional is (6).

(6) "If the patient has malaria, then she has a fever" (Johnson-Laird & Byrne, 2002, p. 663).

The possibilities of (6) are those in (7). As it can be checked, they appear to be equivalent to the ones of (1): the formal structure of (7) seems to match with that of (2).

(7) $P(p \ \& \ q) \ \& \ P(\neg p \ \& \ q) \ \& \ P(\neg p \ \& \ \neg q)$.

Given that it is not possible to have a disease such as malaria without a fever, that scenario cannot be included in (7).

In the case of Enabling, the possibility in the center of (7) changes. Its example is (8).

(8) "If oxygen is present then there may be a fire" (Johnson-Laird & Byrne, 2002, p. 663).

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[1] The following account is based on both the information coming from Table 4 in Johnson-Laird and Byrne (2002) and the description of the ten interpretations in López-Astorga (2020, pp. 40-43). They are indicated in the same order as in López-Astorga (2020). The names of Tautology, Conditional, Enabling, Disabling, Biconditional, Strengthen Antecedent, Relevance, Tollens, Ponens, and Deny Antecedent & Affirm Consequent are introduced by Johnson-Laird and Byrne (2002). Examples (4), (6), (8), (10), (12), (14), (16), (18), (20), and (22) are also taken from Johnson-Laird and Byrne (2002). Those examples are used in López-Astorga (2020) as well, except (12) and (22), which are replaced in the latter paper with deontic conditionals proposed in Johnson-Laird and Byrne (2002) too. Regarding (5), (7), (9), (11), (15), (17), (19), and (21), they are not exactly expressed as in López-Astorga (2020). Letters 'p' and 'q' refer to the clauses here. Nevertheless, in López-Astorga (2020) the clauses are often reproduced. For instance, (5), which corresponds to [V] in López-Astorga (2020), is expressed there as follows: "Possible (there are lights over there & there is a road) & Possible (there are lights over there & there is no a road) & Possible (there are not lights over there & there is a road) & Possible (there are not lights over there & there is not a road)" (López-Astorga, 2020, p. 40).

What is impossible now is the existence of a fire without the existence of oxygen. Accordingly, the possibilities of (8) are (9).

(9) P(p & q) & P(p & ¬q) & P(¬p & ¬q).

As far as Disabling is concerned, one example can be (10).

(10) "If the workers settle for lower wages then the company may still go bankrupt" (Johnson-Laird & Byrne, 2002, p. 663).

If (10) is correct, one cannot imagine a situation in which the wages keep being the same and the company survives. Therefore, its possible circumstances are those in (11).

(11) P(p & q) & P(p & ¬q) & P(¬p & q).

The next three interpretations only admit two combinations. They are Biconditional, Strengthen Antecedent, and Relevance.

Biconditional matches with the logical biconditional: it is true when its two clauses are, or none of them is; it is false when just one of the clauses is true. The instance is (12).

(12) "If he drives the car then he will crash it" (Johnson-Laird & Byrne, 2002, p. 663).

If the communicative intention of the person transmitting (12) is taken into account, it is obvious that the sentence means that it is not possible that he drives and he does not crash the car, or that he crashes the car without driving it. Hence, the possibilities are (13).

(13) P(p & q) & P(¬p & ¬q).

Regarding Strengthen Antecedent, the example Johnson-Laird and Byrne presents is the following:

(14) "If there is gravity (which there is) then your apples may fall" (Johnson-Laird & Byrne, 2002, p. 663).

There is gravity. So, the cases in which there is not must be eliminated. The result is (15).

(15) P(p & q) & P(p & ¬q).

The factual example of Relevance is (16).

(16) "If you are interested in seeing Vertigo then it is on TV tonight" (Johnson-Laird & Byrne, 2002, p. 663).

That Vertigo is on TV tonight is, according to (16), something that cannot be hesitated. So, the scenarios in which that does not happen are not valid. Only two possibilities remain:

(17) P(p & q) & P(¬p & q).

The three last interpretations have to do with irony, figurative language, and pragmatics. They are Tollens, Ponens, and Deny Antecedent & Affirm Consequent. An example for Tollens can be (18).

(18) "If it works then I'll eat my hat" (Johnson-Laird & Byrne, 2002, p. 663).

This sentence means that it is impossible that it works and, accordingly, that it is also impossible that the speaker's hat is eaten. Therefore, its only possibility is (19).

(19) P(¬p & ¬q)

On the other hand, the instance of Ponens can be (20).

(20)   "If my name is Alex then Viv is engaged" (Johnson-Laird & Byrne, 2002, p. 663).

As (18), (20) refers to two facts. In this case, the facts are: the speaker's name is Alex and Viv is engaged. Hence, again, only one possibility can be linked to (20). It is (21).

(21) P(p & q).

Lastly, the example of Deny Antecedent & Affirm Consequent is an irony:

(22)   "If Bill Gates needs money then I'll lend it to him" (Johnson-Laird & Byrne, 2002, p. 663).

What (22) expresses is also obvious: although he does not need money, the speaker is going to give it to Bill Gates. Thereby, the possibility in this final interpretation is (23).

(23)   P(¬p & q).

According to Johnson-Laird and Byrne (2002), there are not more interpretations of the conditional. However, before continuing, several points should be mentioned. On the one hand, (5), (7), (9), (11), (13), (15), (17), (19), (21), and (23) are sets of combinations indicated in Table 4 in Johnson-Laird and Byrne (2002). Nevertheless, they appear in that table in another way: as presented in previous versions of the theory of mental models. The present paper has resorted to a way more or less akin to the one many times used in the updated versions of the theory (see, e.g., Khemlani, Hinterecker, & Johnson-Laird, 2017). Besides, this classification has also been deployed in a lot of works with diverse purposes (e.g., López-Astorga, 2020, where, (4), (6), (8), (10), (14), (16), (18), and (20) are literally cited too). This shows the relevance and the potential for general linguistic analyses about the conditional of the classification. So, it justifies to keep using it. That will be done in the present paper, but adopting a different perspective. As indicated, the point of view will be here that of Chrysippus' criterion. The ten interpretations of the conditional will be deemed as types of conditionals and reviewed under the requirement Chrysippus of Soli gave. Thus, the goal is to check whether or not those ten kinds of conditionals follow Chrysippus' rule and, therefore, can be considered as correct conditionals under his logical framework. Before starting the review, the next section explains in more details what Chrysippus' criterion is.

The criterion: the need for a connection between the two clauses

The criterion Chrysippus of Soli proposed is related to what in logic is named 'strict implication' (see, e.g., Lenzen, 2019). There have been several attempts to capture it by means of a logical formula. (24) is the formula usually expressing the material conditional relation, that is, the conditional relation understood in the same manner as Philo of Megara.

(24) $p \rightarrow q$

Where '$\rightarrow$' means conditional relation.

The logical form corresponding to (1) is (24). In the same way, (24) could also be applied to sentences (4), (6), (8), (10), (12), (14), (16), (18), (20), and (22). Nonetheless, one might think that, if a sentence fulfills Chrysippus' criterion (i.e., the negation of its consequent and its antecedent cannot be true at the same time), it should have a different formula making that criterion clear. A try in this regard was the one of McCall (1975). However, as far as the aims of this paper are concerned, perhaps a more interesting formula can be the one Lenzen (2019) presents. Lenzen's (2019) formula is offered after analyzing approaches such as those of McCall and Leibniz. With not exactly the same symbols, the formula is (25) (see Lenzen, 2019, p. 548).

(25) $(p \rightarrow q) \leftrightarrow \neg \Diamond (p \wedge \neg q)$

Where '$\leftrightarrow$' stands for biconditional relation, '$\Diamond$' is the modal operator of possibility, '$\neg$' represents now logical negation, and '$\wedge$' refers to logical conjunction.

What is interesting about (25) is that it imposes an important limitation to (24). The conditional relation can only happen if a requirement is fulfilled: there cannot be a possible world, or a state-description in the sense Carnap (1947) gives this expression, in which the conjunction $p \wedge \neg q$ holds. This is because the logical biconditional ($\leftrightarrow$) is, as it is well known, an operator establishing the conditional relation in the two directions. That is what (26) shows.

(26) $(p \leftrightarrow q) = [(p \rightarrow q) \wedge (q \rightarrow p)]$

Thus, the biconditional relation means equivalence.

On the other hand, '$\Diamond$' indicates the situation in which there is a state-description in which the formula following that symbol is true. If 'state-description' is understood in the sense Carnap (1947) attributes to it, it can be thought that a state-description consists of every atomic formula that can be taken into account. The difference between state-descriptions has to do with the number of those atomic formulae that are negated in each of them. In other words, every state-description includes every atomic formula or its negation, it is consistent (it does not contain a formula and its negation at once), and there are not two identical state-descriptions.

This paper is intended to check whether or not the ten interpretations of the conditional identified by the theory of mental models (deemed as kinds of conditionals) comply Chrysippus' requirement. Therefore, the purpose is to verify whether or not they are coherent with (25). The next section reviews this.

The requirement and the ten types of conditionals

It is not the first time Chrysippus' criterion is related to the theory of mental models (e.g., López-Astorga, 2016). Nevertheless, the idea now is not to detect commonalities between Stoic logic (represented by Chrysippus) and the aforementioned theory. As stated, the main goal here is to review whether or not the ten interpretations described above follow Chrysippus' requirement. The interpretations will be deemed as kinds of conditionals below. The reason is that, because the theory of mental models tries to describe human thought, the present paper seeks to delve into the way the human mind understands the conditional.

The first point to work in this way can be to take into account that the theory of mental models has been linked to modal logic too (e.g., López-Astorga, 2021). The idea has been to consider the possibilities of the theory as possible worlds of modal logic. Thus, for example, it can be thought that the three possibilities in (7) are three state-descriptions. In one of them, the patient has both malaria and a fever. In the second one, she has a fever without malaria. In the last one, the patient does not have malaria or a fever. If this is in this manner, (25) allows affirming that all of the kinds of conditionals in which $p \land \neg q$ is not possible are consistent with Chrysippus' criterion. A type of conditional for which the combination p & $\neg q$ is not a possibility is a type for which a state-description in which $p \land \neg q$ is true is not possible, as provided by the second clause in (25).

As (7), (13), (17), (19), (21), and (23) show, that is what occurs in Conditional, Biconditional, Relevance, Tollens, Ponens, and Deny Antecedent & Affirm Consequent. So, it can be claimed that those six interpretations fulfill (25) and, accordingly, Chrysippus' requirement.

Since the other four types of conditionals include the combination p & $\neg q$, in principle, it could be thought that they are inconsistent with Chrysippus' criterion. Nevertheless, this is not the case. Ignoring (25), the logical form of those four kinds (Tautology, Enabling, Disabling, and Strengthen Antecedent) is not initially (24), but (27).

(27) $p \rightarrow \Diamond q$

This is because Tautology, Enabling, Disabling, and Strengthen Antecedent have a modal verb in their consequents ('may'). Hence, those consequents do not express facts, but just possibilities. Thereby, the application of (25) to (27) leads to (28).

(28) $(p \rightarrow \Diamond q) \leftrightarrow \neg\Diamond(p \land \neg\Diamond q)$

Therefore, what Chrysippus' criterion requires for Tautology, Enabling, Disabling, and Strengthen Antecedent is what the second clause in (28) indicates, that is, (29).

(29) $\neg\Diamond(p \land \neg\Diamond q)$

What (29) means is that p cannot be true without the possibility of q being real. In other words, (29) states that there cannot be a state-description in which p is true and q is not possible. So, it is not necessary that q is true when p is, but only that q is possible when p is true. This circumstance seems to happen in Tautology, Enabling, Disabling, and Strengthen Antecedent. There are cases of p & $\neg q$ in these four interpretations. Nonetheless, there are also cases of p & q in all of them. Given that, as said, what (29) asks for is not that q is true provided that p is, but that the possibility of q

being true exists provided that p is, a state-description with p ∧ ¬q is not a problem. That is not a problem because a state-description with p ∧ ¬q is not inconsistent with (29) if there is at least another state-description with p ∧ q at once. And this last condition is fulfilled in Tautology, Enabling, Disabling, and Strength Antecedent. As pointed out, the possibility of p & q existing is in these four kinds of conditionals. Thus, there are state-descriptions with p ∧ ¬q and state-descriptions with p ∧ q. That implies that, when p happens, q is not necessarily true, but it is possible.

Accordingly, given that it is possible que p is true and q is possible at the same time, Tautology, Enabling, Disabling, and Strengthen Antecedent are also coherent with the requirement Chrysippus of Soli indicated. The ten kinds of conditionals derived from the interpretations Johnson-Laird and Byrne (2002) presented fulfill that criterion.

Conclusions

What the theory of mental models proposes with its ten interpretations is to show the ten ways a human being can understand a conditional. From this point of view, the present paper offers an additional datum: the possible interpretations of the conditional, which can be deemed as kinds of conditionals, follow the criterion indicated by Chrysippus of Soli. Thereby, it can be stated that human beings tend to understand conditionals as strict implications.

Against this, it can be argued that there are cases of conditionals such as (1). If (1) is true, its possibilities are (2). If those possibilities are considered as state-descriptions, they fulfill (25): the conjunction p ∧ ¬q happens in none of them. However, as said, people reject sentences such as (1). The reason for this is that they are not actually consistent with Chrysippus' requirement. There are not semantic or pragmatic causes preventing a state-description in which elephants can sing and groundhogs cannot drive. This is because, apart from what general knowledge provides for elephants and groundhogs, the meanings of singing and driving are not linked, and it is possible to do one of these actions without doing the other one. In addition, again, apart from what general knowledge provides for elephants and groundhogs, in principle, pragmatics indicates nothing about the truth or falsity of the clauses. Therefore, in a narrow sense, (1) does not fulfill (25). The denial of its consequent is not incompatible with its antecedent. From the linguistic perspective, those facts do not have an influence on each other.[2] Accordingly, it seems that individuals accept conditionals of different kinds (following the theory of mental models, its ten interpretations). However, all of those types have a common feature: they are coherent with the manner Chrysippus of Soli understood the conditional.

The consequences of all this can be analyzed from two points of view. On the one hand, it is further support for the theory of mental models: it gives one more characteristic to the list of interpretations of the conditional, which appears to make that list even more plausible. On the other hand, in a way consistent with the literature (e.g., López-Astorga, 2016), it shows that to keep addressing theories from the past in general and the one of Chrysippus of Soli in particular is relevant. So, following that very literature, it seems appropriate to continue to use criteria such as

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[2] This is very different from what occurs, for example, in (6), where there is a clear relation between having malaria and having a fever. It is also different from cases such as the one of (18), where pragmatics lead to only one scenario: that in which it does not work and the hat is not eaten.

the one Chrysippus presented for the conditional in studies akin to that described in the present paper.

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