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Research article

Induction: Possibilities, frequency, and confidence

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Abstract

The theory of mental models describes how human beings often make inductive inferences. That account is based on possibilities. It claims that, in inductions, people tend to prefer the possibility that seems to be more probable. The present paper tries to develop the account by proposing an additional quantitative method to determine which of the two possibilities in inductive processes is preferable. The paper resorts to a Non-Axiomatic Logic: NAL. NAL assigns quantitative values to the frequency and confidence of sentences. It also shows how those values can be transmitted from premises to conclusions in inductive inferences. This part of NAL is applied to the account of induction the theory of mental models gives.

Keywords: confidence, frequency, induction, mental models, non-axiomatic logic

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Introduction

The theory of mental models (e.g., Johnson-Laird, 2023) has presented a way to explain how human beings make inductions (see also, e.g., Johnson-Laird, 2012). The thesis of the theory is that people make inferences such as (1) because they consider those inferences to be more probable than inferences such as (2).

(1) A person has drunk three cups of coffee. Therefore, that person will be awake tonight.

(2) A person has drunk three cups of coffee. Therefore, that person will not be awake tonight.

The theory is based on possibilities. The idea is that a possible scenario in which a person has three cups of coffee and that person does not sleep that night is more probable than a possible scenario in which that very person sleeps that night after drinking three cups of coffee (explanations of induction in the theory of mental models using different examples are to be found in, e.g., Johnson-Laird, 2012; López-Astorga, 2022).

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This account can be developed to include a quantitative method to decide which of the two possibilities is more probable. The theory of mental models does not ignore the need to define the concept of probability (e.g., Khemlani & Johnson-Laird, 2022). However, in cases such as those of (1) and (2), an additional quantitative method can be possible.

To offer that quantitative method, the present paper will try to link the theory of mental models to NAL (Non-Axiomatic Logic; e.g., Wang, 2013). NAL is a non-axiomatic logic with an important characteristic: it adopts the Assumption of Insufficient Knowledge and Resources (AIKR; see also, e.g., Wang, 2011). This means that the system has gaps in knowledge because a lot of information is missing. NAL quantitatively explains inductive processes. It assigns two numbers to each of the sentences involved in the inference. One of them is the frequency of the sentence. The second number is the confidence of the sentence.

To try to introduce quantitative values of frequency and confidence into the framework of the theory of mental models is what will be done below. That will provide a criterion to decide between the two possibilities in induction processes. The possibilities with higher values of frequency and confidence should be preferred.

The first section will show how the theory of mental models understands human induction processes. The second section will explain the NAL proposal for induction. The last section will attempt to link both approaches to give the theory of mental models a quantitative method to make decisions about possible conclusions in inductive inferences.

Induction using mental models

According to the theory of mental models, when human beings digest linguistic information, they consider the possible circumstances to which that information refers (see also, e.g., Khemlani, Byrne, & Johnson-Laird, 2018). For example, one can think that (1) refers to possibilities (3) to (5).

(3) $\Delta(D \wedge A)$

(4) $\Delta(\neg D \wedge A)$

(5) $\Delta(\neg D \wedge \neg A)$

In (3), (4), and (5) ‘Δ’ indicates that what follows between brackets is a possibility. The usual symbol in modal logic is not used here because the theory of mental models is not a modal logic (e.g., Johnson-Laird & Ragni, 2019). ‘Δ’ represents what ‘possibility’ often means in natural language (for the meaning of ‘possibility’ in the theory of mental models, see, e.g., Khemlani et al., 2018). ‘D’ stands for the fact that the person drank three cups of coffee. ‘A’ expresses the fact that the very person is awake. ‘$\wedge$’ denotes conjunction. ‘$\neg$’ is the symbol pointing out negation.

Possibilities (3) to (5) are the possibilities that can be associated with (1) if deemed as a true inference. (3) describes the case in which a person has three cups of coffee and is awake. (4) represents the situation in which a person does not have three cups of coffee but the person is awake. (5) refers to the situation in which the person does not drink coffee or is awake.

In the theory of mental models, the possibilities are joined in a conjunctive manner (see also, e.g., Khemlani, Hinterecker, & Johnson-Laird, 2017). Therefore, (3), (4), and (5) should be considered as conjuncts in a conjunction such as (6).

(6) $\Delta(D \wedge A) \wedge \Delta(\neg D \wedge A) \wedge \Delta(\neg D \wedge \neg A)$

The structure of (6) allows thinking about a conditional such as (7) (for the possibilities corresponding to the conditional in the theory of mental models, see, e.g., Byrne & Johnson-Laird, 2020).

(7) If a person has drunk three cups of coffee, that person will be awake tonight

Actually, in the original explanations from the theory of mental models (e.g., Johnson-Laird, 2012), an inference such as (1) is not related to a conditional such as (7). The relation is between an inference such as (1) and a conditional taking the conclusion in (1) as its antecedent and the premise in (1) as its consequent. So, the conditional would be akin to 'If a person is awake tonight, that person drank three cups of coffee'. But, as it can be noted from the account below, if more papers supporting the theory are considered, (7) can also be supposed for an induction such as (1). The arguments in this section will show that the reasons for this are trivial and obvious.

The point is that (7) allows transforming inference (1) into an application of Modus Ponendo Ponens. The premise in (1) is the antecedent in (7). It enables us to derive the consequent in (7). This seems to reveal that the theory of mental models depends on classical propositional calculus. But it does not. The process through which Modus Ponendo Ponens is used in the theory of mental models is the following.

Given that the premise in (1) establishes that the person has had three cups of coffee, the statement of the premise removes the second and third conjuncts in (6), that is, it removes (4) and (5). The reason for this is that the person does not drink three cups of coffee in (4) or (5). Only one possibility remains: the first one in (6), that is, (3). In scenario (3) described, the person is awake (for the way the theory of mental models explains the natural human use of Modus Ponendo Ponens, see also, e.g., Byrne & Johnson-Laird, 2009).

There are many more differences between the theory of mental models and propositional calculus. As far as this paper is concerned, it is important to highlight that the negation of conditionals in the former is not as in classical logic (e.g., López-Astorga, Ragni, & Johnson-Laird, 2022). This can be seen by considering the possibilities corresponding to the case in which (2) is true, and hence (1) is false. Those possibilities are (8), (4), and (5).

(8) $\Delta(D \wedge \neg A)$

So, the possibilities to which (2) refers are those in conjunction in (9).

(9) $\Delta(D \wedge \neg A) \wedge \Delta(\neg D \wedge A) \wedge \Delta(\neg D \wedge \neg A)$

This allows checking that, unlike propositional calculus, the theory of mental models negates conditionals such as (7) in the way expressed in (10).

(10) If a person has drunk three cups of coffee, that person will not be awake tonight

The theory of mental models proposes that the negation of a conditional can consist of the negation of its consequent (see also, e.g., Khemlani, Orenes, & Johnson-Laird, 2014). This is the reason why possibilities such as (4) and (5) are deemed in the theory as 'presuppositions': possibilities such as (4) and (5) are admissible both if (7) is the case (i.e., the conditional is the case) and if (10) is the case (i.e., the conditional is not the case) (for the concept of 'presupposition' in the theory of mental models, see also, e.g., Espino, Byrne, & Johnson-Laird, 2020). This reveals that the difference between (1) and (2) is just the difference between (3) and (8). (1) and (2) share the rest of the possibilities, that is, (4) and (5). What the theory of mental models provides is that individuals accept inference (1) because they think that (3) is more probable than (8). At least, this is what one can derive from different papers supporting the theory.

The question arising is about the best manner to describe the decision process leading to assume that (3) is preferable over (8). NAL can give a quantitative criterion to be incorporated into the theory.

Induction, frequency, and confidence

NAL is a logic to use when AIKR is followed. It has development levels from NAL-1 to NAL-9. The differences between those levels are in their complexity degrees and their numbers of inference rules. The higher the level is, the higher its complexity degree and its number of inference rules are (Wang, 2013). NAL-1 suffices for the aims of this paper. Accordingly, the characteristics this section will assign to NAL are already present in NAL-1 (the description of those characteristics will be based on the account of NAL-1 in Wang, 2013).

In NAL, the relations between terms are inheritance relations. They can be expressed such as the relation in (11).

(11) $S \rightarrow P$

In (11), 'S' stands for the subject, 'P' represents the predicate, and '$\rightarrow$' is not the symbol for the conditional in classical logic. It is the symbol for inheritance relations (see, e.g., Wang, 2013, Definition 2.2, p. 14). Therefore, (11) indicates that S and P are linked by an inheritance relation. The relation is from S to P. Inheritance relations are understood using the concepts of extension and intension. Wang (2013, Definition 2.8, p. 18) establishes that 'the extension of a term T' includes all of the terms x such that (12) holds.

(12) $x \rightarrow T$

And 'the intension of a term T' includes all of the terms x such that (13) holds.

(13) $T \rightarrow x$

Given, for example, (14),

(14) *Pig* $\rightarrow$ *Mammal*

'Pig' is part of the extension of 'Mammal', and 'Mammal' is part of the intension of 'Pig'.

Because the system works in a context in which AIKR is admitted, it is not sure about the inheritance relations accepted. For this reason, the system assigns to each of the inheritance relations a frequency and a confidence. It does that considering negative and positive evidence for inheritance relations. NAL identifies positive and negative evidence by the concepts of extension and intension (Wang, 2013, Definition 3.1, p. 27). However, for the goals of the present paper, which will try to adapt NAL to the theory of mental models, it is enough to define positive and negative evidence in their usual senses in science: what confirms a sentence and what refutes a sentence, respectively. The paper will respect the definitions of frequency and confidence NAL provides (they are in Wang, 2013, Definition 3.3, p. 29):

Definition of frequency: $f =_{df} w^{+}/w$

In this definition, 'f' refers to 'frequency', 'w^{+}' is the number for positive evidence, and 'w' is the number for total evidence, that is, the sum of positive evidence plus negative evidence (the latter idea is in Wang, 2013, Definition 3.2, p. 28).

Definition of confidence: $c =_{df} w/(w + k)$

In the definition of confidence, 'c' stands for 'confidence' and 'k' is a constant. Usually, k = 1 (Wang, 2013, p. 30).

This allows for making the certainty degrees in inheritance relations explicit. For instance, 50 pigs have been seen and those 50 pigs are mammals. With these data, f = 50/50 = 1 and c = 50/(50 + 1) = 50/51 = 0.98. In this way, (14) can be expressed as in (15).

(15) $\quad Pig \rightarrow Mammal$ (f, c) = $Pig \rightarrow Mammal$ (1, 0.98)

The values of frequency and confidence are always between 0 and 1. 0 is the lowest level for both frequency and confidence. Hence, 1 is the highest level for both frequency and confidence.

As far as induction is concerned, the schema in NAL is as follows (the schema is in Wang, 2013, p. 53):

(16) $\quad M \rightarrow P\ (f_1, c_1)$

$M \rightarrow S\ (f_2, c_2)$

Therefore, $S \rightarrow P\ (f_3, c_3)$

Wang (2013) presents formulae to calculate f_3 and c_3. The formulae are these (they are in Wang, 2013, Table 4.7, p. 61):

For f_3, $f_3 = f_1$

For c_3, $c_3 = (f_2 * c_1 * c_2)/[(f_2 * c_1 * c_2) + k]$

Let the sentences in (17) be the premises of an inference.

(17) $\quad Pig \rightarrow Pink$ (0.8, 0.98)

$Pig \rightarrow Mammal$ (1, 0.98)

The values assigned to the first premise (f = 0.8 and c = 0.98) are based on a hypothetical scenario in which only 40 of the 50 pigs seen are pink, the other 10 pigs being different colours.

Applying the formulae above for induction, (18) can be derived from (17).

(18) *Mammal* → *Pink* (0.8, 0.49)

Where $f_3 = f_1 = 0.8$ and $c_3 = (1 * 0.98 * 0.98)/[(1 * 0.98 * 0.98) + 1] = 0.49$.

Perhaps all of this can be adapted to the way the induction processes are carried out within the theory of mental models. The next section will try to do that.

Mental models, frequency, and confidence

For example, five people who have drunk three cups of coffee can be supposed to. This would lead to (19).

(19) *Person* → *Three cups of coffee* (1, 0.83)

Where $f = 5/5 = 1$ and $c = 5/(5+1) = 5/6 = 0.83$.

Likewise, three of those five people can be assumed to be awake tonight. (20) and (21) would capture this circumstance.

(20) *Person* → *Awake* (0.6, 0.83)

Where $f = 3/5 = 0.6$ and $c = 5/(5 + 1) = 5/6 = 0.83$.

(21) *Person* → *Not awake* (0.4, 0.83)

Where $f = 2/5 = 0.4$ and $c = 5/(5 + 1) = 5/6 = 0.83$.

The observation of the five people (all of them drinking three cups of coffee, three of them being awake tonight, and two of them not being awake tonight) allows making at least two inferences. In the first one, the premises would be (20) and (19). Those premises would enable us to come to an inductive conclusion (22).

(22) *Three cups of coffee* → *Awake* (0.6, 0.41)

Where $f_3 = f_1 = 0.6$ and $c_3 = (1 * 0.83 * 0.83)/[(1 * 0.83 * 0.83) + 1] = 0.41$.

In the second inference, the premises would be (21) and (19). The inductive conclusion would be (23) in this case.

(23) *Three cups of coffee* → *Not awake* (0.4, 0.41)

Where $f_3 = f_1 = 0.4$ and $c_3 = (1 * 0.83 * 0.83)/[(1 * 0.83 * 0.83) + 1] = 0.41$.

So, in the scenario assumed (five people drinking three cups of coffee, three of them sleeping tonight, and two of them not sleeping tonight), (22) should be preferred over (23). Both (22) and (23) have the same confidence (0.41). However, the frequency of (22) is higher than that of (23) (0.6 versus 0.4). Given AIKR, these numbers can change, and with them the preference of (22) over (23). But this is one of the strengths of NAL, which helps it simulate the real situations in which inductions are carried out with just the information available in those moments. With the information supposed here, (22) is preferable.

The point now is how to relate this to the theory of mental models. The relation should be provided between (22) and (7) (i.e., between (22) and 'If a person has drunk three cups of coffee, that person will be awake tonight') and between (23) and (10) (i.e., between (23) and 'If a person has drunk three cups of coffee, that person will not be awake tonight'). It is important to keep in mind that to relate (22) to (7) is mainly to relate (22) to (3), that is, to $\Delta(D \wedge A)$. The other possibilities corresponding to (7) are presuppositions. In the same way, to relate (23) to (10) is chiefly to relate (23) to (8), that is, to $\Delta(D \wedge \neg A)$. In this case, the other possibilities corresponding to (10) are also presuppositions.

One might think that those relations are not legitimate. (7) and (10) are conditionals, and (22) and (23) represent inheritance relations. Regarding this, several points can be made. As understood by the theory of mental models, (7) and (10) are not logical conditionals. Therefore, they are not material conditionals either. They are only conditionals expressed in a natural language, in this case, English, linking what is said in the antecedent to what is indicated in the consequent. On the other hand, the aim of the present paper is not strictly to follow NAL. The paper is intended to take components of NAL that can help give the theory of mental models an additional quantitative method to capture why some possibilities are preferred over other possibilities in inductions. In the process, those components of NAL can be adapted as required.

What is important is that the antecedent of (7) expresses the same as the subject of (22) and that the consequent of (7) represents the same situation as the predicate of (22). Likewise, the antecedent of (10), which matches the antecedent of (7), transmits the same information as the subject of (23), and the consequent of (10) refers to the same semantic content as the predicate of (23). Accordingly, the relation between, on the one hand, (7) and (22), and, on the other hand, (10) and (23) can be justified. If this is correct, the preference of (22) over (23) means the preference of (7) over (10). But, if (7) is the implicit conditional, (1), and not (2), is the correct Modus Ponendo Ponens inference? The premise in (1), that is, 'A person has drunk three cups of coffee', matches the antecedent of (7). As explained above, although the theory of mental models is not propositional calculus, that Modus Ponendo Ponens inference is acceptable under its framework. The conjunction of possibilities (6), that is, $\Delta(D \wedge A) \wedge \Delta(\neg D \wedge A) \wedge \Delta(\neg D \wedge \neg A)$, is that corresponding to (7). The premise (1) is true only in the first conjunct in (6), that is, in (3), that is, in $\Delta(D \wedge A)$. In that conjunct, the person is awake tonight.

Conclusions

The theory of mental models is intended to offer an account of the inferential processes people do. The particular case of inductive inferences has been analyzed in the present paper. According to the theory, inductive inferences are made by implicit conditionals. Those implicit conditionals are linked to the possibilities individuals deem as the most probable scenarios. The purpose of this paper has been to give a quantitative account of the choice of possible scenarios in induction. That account has been based on NAL, a non-axiomatic logic accepting AIKR.

In NAL, there are inheritance relations between subjects and predicates. NAL attributes quantitative values to those inheritance relations to indicate their frequency and confidence. The inductive conclusions with higher values are the conclusions that should be preferred.

The account above has explained how these components of NAL can be adapted and applied to the inductive processes in the theory of mental models. The possibility and hence, ignoring presuppositions, the conditional to be chosen is the possibility and the conditional that can be linked to the inheritance relation with the highest frequency and confidence values.

Just one example has been addressed here. That example is illustrative enough to describe how the application of NAL to the theory of mental models can be. Both NAL and the theory of mental models refer to more kinds of inferences (e.g., deduction and abduction). Arguments similar to those introduced for induction here can also be used in the cases of the other types of inferences both frameworks deal with.

Declaration of Conflicts of Interests

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